

Question 9

Find a Maclaurin series for $1/(2 - x)$ by first factoring the denominator. State the radius and interval of convergence, including endpoint tests.

Question 9 – Solution

Step 1: Put the function in geometric form.

$$\frac{1}{2-x} = \frac{1}{2} \frac{1}{1-x/2}.$$

Using the geometric identity with $r = x/2$ gives

$$\frac{1}{2-x} = \frac{1}{2} \sum_{n=0}^{\infty} \left(\frac{x}{2}\right)^n = \sum_{n=0}^{\infty} \frac{x^n}{2^{n+1}}.$$

Step 2: Translate the restriction.

The condition $|x/2| < 1$ becomes $|x| < 2$, so $R = 2$.

Step 3: Test endpoints.

At $x = 2$, each term equals $1/2$. At $x = -2$, the terms alternate between $\pm 1/2$. Neither tends to zero, so both series diverge.

Conclusion.

The interval is $\boxed{(-2, 2)}$.