

Question 7

Find a Maclaurin series for $1/(1-x^2)$ by recognizing a geometric series. Explain why only even powers occur and determine the interval of convergence.

Question 7 – Solution**Step 1: Identify the ratio.**Apply $1/(1-r) = \sum r^n$ with $r = x^2$:

$$\frac{1}{1-x^2} = \sum_{n=0}^{\infty} (x^2)^n = \sum_{n=0}^{\infty} x^{2n}.$$

Only even powers occur because each power of the ratio x^2 adds two to the exponent.**Step 2: Translate the geometric restriction.**

$$|x^2| < 1 \iff |x| < 1.$$

Hence $R = 1$.**Step 3: Test the endpoints.**At both $x = 1$ and $x = -1$, every term x^{2n} equals 1. Thus the series diverges at both endpoints.**Conclusion.**The interval is $\boxed{(-1, 1)}$.