

Question 6

Use the differentiated geometric identity to find a power series for $x/(1-x)^2$. State the index, radius, and exact interval of convergence.

Question 6 – Solution

Step 1: Differentiate the geometric series.

For $|x| < 1$,

$$\frac{1}{(1-x)^2} = \sum_{n=1}^{\infty} nx^{n-1}.$$

Step 2: Multiply by x .

$$\boxed{\frac{x}{(1-x)^2} = \sum_{n=1}^{\infty} nx^n}.$$

Multiplication by x shifts every exponent upward by one but does not change the radius.

Step 3: Test the boundary.

The radius remains $R = 1$. At $x = 1$, the terms are n ; at $x = -1$, they are $n(-1)^n$. Neither term sequence approaches zero.

Conclusion.

The interval of convergence is $\boxed{(-1, 1)}$.