

Question 5

Derive the Maclaurin series for $\arctan x$ by integrating the series for $1/(1+x^2)$. Explain the odd powers and determine the endpoint behavior.

Question 5 – Solution**Step 1: Expand the derivative.**With geometric ratio $-t^2$,

$$\frac{1}{1+t^2} = \sum_{n=0}^{\infty} (-1)^n t^{2n}, \quad |t| < 1.$$

Step 2: Integrate from 0 to x .

$$\arctan x = \int_0^x \frac{dt}{1+t^2} = \boxed{\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1}}, \quad |x| < 1.$$

Integrating the even powers t^{2n} produces only odd powers x^{2n+1} ; the value at 0 fixes the constant.**Step 3: Test endpoints.**At $x = 1$, the alternating series converges to $\pi/4$. At $x = -1$, it converges to $-\pi/4$. Both are conditional.**Conclusion.**The radius is $R = 1$ and the interval is $\boxed{[-1, 1]}$.