

Question 4

Derive a power series for $\log(1 + x)$ by integrating a geometric series. Determine the constant of integration and the exact interval of convergence.

Question 4 – Solution**Step 1: Expand the derivative.**For $|t| < 1$,

$$\frac{1}{1+t} = \sum_{n=0}^{\infty} (-1)^n t^n.$$

Step 2: Integrate from 0 to x .Term-by-term integration is valid when $|x| < 1$:

$$\begin{aligned} \log(1+x) - \log 1 &= \int_0^x \frac{dt}{1+t} \\ &= \sum_{n=0}^{\infty} (-1)^n \frac{x^{n+1}}{n+1}. \end{aligned}$$

Thus

$$\log(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1} x^n}{n}.$$

The lower limit makes the constant 0.

Step 3: Test endpoints.At $x = 1$, the alternating harmonic series converges to $\log 2$. At $x = -1$, the series is $-\sum 1/n$ and diverges.**Conclusion.** $R = 1$ and the interval is $\boxed{(-1, 1]}$.