

Question 3

Differentiate the geometric series term by term to find a power series for $1/(1-x)^2$. Reindex it in powers x^n and state its full interval of convergence.

Question 3 – Solution**Step 1: Begin inside the radius.**For $|x| < 1$,

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n.$$

A power series may be differentiated term by term at every point strictly inside its radius.

Step 2: Differentiate.

$$\frac{d}{dx}(1-x)^{-1} = (1-x)^{-2} = \sum_{n=1}^{\infty} nx^{n-1}.$$

Let $k = n - 1$:

$$\boxed{\frac{1}{(1-x)^2} = \sum_{k=0}^{\infty} (k+1)x^k}.$$

Step 3: Check the boundary.Differentiation preserves the radius $R = 1$, but endpoints must be retested. At $x = 1$ the terms are $k + 1$; at $x = -1$ their magnitudes are $k + 1$. Neither tends to zero.**Conclusion.**The interval is $\boxed{(-1, 1)}$.