

Question 2

Obtain a power series centered at 0 for $f(x) = 1/(1+x)$ by substituting into the geometric series. State its radius and interval of convergence and test both endpoints directly.

Question 2 – Solution

Step 1: Make the sign substitution.

Replace x by $-x$ in $1/(1-x) = \sum x^n$:

$$\frac{1}{1+x} = \sum_{n=0}^{\infty} (-x)^n = \sum_{n=0}^{\infty} (-1)^n x^n.$$

Step 2: Translate the restriction.

The geometric condition is $|-x| < 1$, equivalent to $|x| < 1$, so $R = 1$.

Step 3: Test the endpoints.

- At $x = 1$, the series is $\sum (-1)^n$, which diverges by the nth-term test.
- At $x = -1$, it is $\sum 1$, which diverges.

Conclusion.

The interval is $\boxed{(-1, 1)}$. Although $1/(1+x)$ is finite at $x = 1$, its Taylor series about 0 does not converge there.