

Question 1

Starting from the geometric-series formula, find a power-series representation centered at 0 for $f(x) = 1/(1 - x)$. State its radius and interval of convergence, including endpoint tests.

Question 1 – Solution

Step 1: Use the geometric prototype.

For a geometric series,

$$\sum_{n=0}^{\infty} r^n = \frac{1}{1-r} \quad (|r| < 1).$$

Set $r = x$ to obtain

$$\boxed{\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n}.$$

Step 2: Carry over the convergence condition.

The requirement $|r| < 1$ becomes $|x| < 1$, so the radius is $R = 1$.

Step 3: Test the endpoints.

At $x = 1$, the series is $\sum 1$; at $x = -1$, it is $\sum (-1)^n$. In both cases the terms fail to approach zero.

Conclusion.

The interval is $\boxed{(-1, 1)}$. The identity holds only there, even though the rational function is defined at some points outside that interval.