

Question 10

Find the interval of convergence and sum function of $\sum_{n=0}^{\infty} (x^2)^n$. Treat it as a geometric series in x^2 and test $x = \pm 1$.

Question 10 – Solution**Step 1: Apply the geometric criterion.**

The common ratio is $r = x^2$. Therefore

$$|r| < 1 \iff |x^2| < 1 \iff x^2 < 1 \iff |x| < 1.$$

Step 2: Test the endpoints.

At both $x = 1$ and $x = -1$, we have $x^2 = 1$, so the series becomes $\sum 1$ and diverges.

Step 3: Find the sum.

For $|x| < 1$, the geometric formula gives

$$\sum_{n=0}^{\infty} (x^2)^n = \frac{1}{1 - x^2}.$$

Conclusion.

The interval of convergence is $\boxed{(-1, 1)}$. As a power series in x , only even powers occur, but its center is still 0 and its radius is 1.