

Question 9

Find the center, radius, and interval of convergence of $\sum_{n=1}^{\infty} \frac{n^2(x-3)^n}{2^n}$. Explain the different roles of the exponential and polynomial factors.

Question 9 – Solution**Step 1: Apply the Ratio Test.**

$$\left| \frac{a_{n+1}}{a_n} \right| = \left(\frac{n+1}{n} \right)^2 \frac{|x-3|}{2} \longrightarrow \frac{|x-3|}{2}.$$

Thus $|x-3| < 2$, giving center $c = 3$, radius $R = 2$, and preliminary interval $1 < x < 5$.

Step 2: Test the endpoints.

- At $x = 5$, the terms reduce to n^2 , so the series diverges.
- At $x = 1$, the terms reduce to $n^2(-1)^n$, whose magnitudes do not approach zero.

Conclusion.

The interval is $\boxed{(1, 5)}$. The exponential coefficient 2^{-n} fixes the radius; the polynomial n^2 has root tending to 1 but makes both boundary terms fail the nth-term test.