

Question 8

Find the center, radius, and interval of convergence of

$$\sum_{n=1}^{\infty} \frac{(-1)^n (x+1)^n}{n}.$$

Track the center carefully and determine which physical endpoint produces the alternating harmonic series.

Question 8 – Solution**Step 1: Find the interior condition.**

The series is centered at $c = -1$. For $a_n = (-1)^n(x+1)^n/n$,

$$\left| \frac{a_{n+1}}{a_n} \right| = |x+1| \frac{n}{n+1} \longrightarrow |x+1|.$$

Thus $|x+1| < 1$, so $-2 < x < 0$ and $R = 1$.

Step 2: Test $x = -2$.

Here $x+1 = -1$, so

$$\frac{(-1)^n(-1)^n}{n} = \frac{1}{n}.$$

The harmonic series diverges; $x = -2$ is excluded.

Step 3: Test $x = 0$.

Here $x+1 = 1$, producing $\sum (-1)^n/n$, which converges conditionally.

Conclusion.

The interval is $\boxed{(-2, 0]}$.