

Question 7

Find the center, radius, and interval of convergence of $\sum_{n=0}^{\infty} (x-2)^n/5^n$. Verify the behavior at both physical endpoints.

Question 7 – Solution

Step 1: Read the geometric structure.

$$\sum_{n=0}^{\infty} \left(\frac{x-2}{5} \right)^n$$

has center $c = 2$ and converges when

$$\left| \frac{x-2}{5} \right| < 1 \iff |x-2| < 5 \iff -3 < x < 7.$$

Thus $R = 5$.

Step 2: Test the endpoints.

- At $x = 7$, the series is $\sum 1$, which diverges.
- At $x = -3$, the series is $\sum (-1)^n$, which also diverges because its terms do not tend to zero.

Conclusion.

The interval is $\boxed{(-3, 7)}$, symmetric about the center 2.