

Question 6

Determine the radius and interval of convergence of $\sum_{n=0}^{\infty} x^n/n!$. Explain why the factorial denominator allows every real x .

Question 6 – Solution

Step 1: Apply the Ratio Test for a fixed x .

$$\left| \frac{x^{n+1}/(n+1)!}{x^n/n!} \right| = \frac{|x|}{n+1} \rightarrow 0.$$

For every fixed real x , the limit is $0 < 1$.

Step 2: State the convergence set.

The series converges absolutely for all $x \in \mathbb{R}$. Thus

$$\boxed{R = \infty}, \quad \boxed{(-\infty, \infty)}.$$

There are no finite endpoints to test.

Step 3: Interpret the factorial.

Each ratio contains the growing divisor $n+1$, which eventually dominates any fixed $|x|$. The series is the familiar expansion $e^x = \sum x^n/n!$.