

Question 5

Determine the radius and set of convergence of $\sum_{n=0}^{\infty} n!x^n$. Use the Ratio Test to show what happens for every nonzero x .

Question 5 – Solution

Step 1: Fix $x \neq 0$ and form the ratio.

With $a_n = n!x^n$,

$$\left| \frac{a_{n+1}}{a_n} \right| = (n+1)|x| \longrightarrow \infty.$$

The Ratio Test therefore gives divergence for every fixed nonzero x . Indeed, the terms eventually increase in magnitude and cannot tend to zero.

Step 2: Check $x = 0$.

At $x = 0$, the series has only its $n = 0$ term nonzero (using $0^0 = 1$ in the power-series convention), so it converges to 1.

Conclusion.

The radius is $\boxed{R = 0}$ and the set of convergence is the single point $\boxed{\{0\}}$. This is not a nondegenerate interval.