

Question 4

Find the radius and interval of convergence of $\sum_{n=1}^{\infty} x^n/n^2$. Explain why its radius matches Question 3 while both endpoints now converge.

Question 4 – Solution

Step 1: Find the radius.

$$\left| \frac{x^{n+1}/(n+1)^2}{x^n/n^2} \right| = |x| \left(\frac{n}{n+1} \right)^2 \rightarrow |x|.$$

Thus the interior condition is $|x| < 1$ and $R = 1$.

Step 2: Test the endpoints.

- At $x = 1$, the series is $\sum 1/n^2$, which converges.
- At $x = -1$, the absolute-value series is again $\sum 1/n^2$, so the series converges absolutely.

Conclusion.

The interval is $\boxed{[-1, 1]}$. Both this series and $\sum x^n/n$ have coefficient roots tending to 1, hence the same radius, but the stronger n^{-2} decay makes both boundary series absolutely convergent.