

Question 3

Find the radius and interval of convergence of $\sum_{n=1}^{\infty} x^n/n$. Identify the harmonic or alternating harmonic series arising at each endpoint.

Question 3 – Solution

Step 1: Find the interior interval.

$$\left| \frac{x^{n+1}/(n+1)}{x^n/n} \right| = |x| \frac{n}{n+1} \longrightarrow |x|.$$

The Ratio Test gives convergence for $|x| < 1$ and divergence for $|x| > 1$, so $R = 1$.

Step 2: Test $x = 1$.

The series becomes $\sum 1/n$, the divergent harmonic series.

Step 3: Test $x = -1$.

The series becomes $\sum (-1)^n/n$, which converges by the Alternating Series Test (but not absolutely).

Conclusion.

The interval is $\boxed{[-1, 1)}$. Convergence at the left endpoint is conditional.