

Question 2

Find the radius and interval of convergence of $\sum_{n=1}^{\infty} \frac{nx^n}{4^n}$. Explain why the polynomial factor n does not change the radius but does determine the endpoint behavior.

Question 2 – Solution**Step 1: Use the Ratio Test.**For $a_n = nx^n/4^n$,

$$\left| \frac{a_{n+1}}{a_n} \right| = \frac{n+1}{n} \frac{|x|}{4} \longrightarrow \frac{|x|}{4}.$$

Convergence requires $|x|/4 < 1$, so $|x| < 4$ and $R = 4$.**Step 2: Test the endpoints.**

- At $x = 4$, the terms are n , which do not approach 0.
- At $x = -4$, the terms are $n(-1)^n$, whose magnitudes grow.

Both endpoint series diverge by the nth-term test.

Conclusion.

The interval is $\boxed{(-4, 4)}$. The factor $(n+1)/n \rightarrow 1$, so n does not alter the radius; at the endpoints it prevents the terms from tending to zero.