

Question 1

Find the center, radius of convergence, and interval of convergence of

$$\sum_{n=0}^{\infty} \frac{x^n}{3^n}.$$

Test $x = -3$ and $x = 3$ separately and identify each resulting endpoint series.

Question 1 – Solution

Step 1: Recognize the geometric series.

$$\sum_{n=0}^{\infty} \frac{x^n}{3^n} = \sum_{n=0}^{\infty} \left(\frac{x}{3}\right)^n.$$

It converges exactly when $|x/3| < 1$, or $|x| < 3$. Thus the center is 0 and $R = 3$.

Step 2: Test the endpoints.

- At $x = 3$, the series is $\sum 1$, which diverges.
- At $x = -3$, the series is $\sum (-1)^n$. Its terms do not approach 0, so it diverges.

Conclusion.

The interval is $\boxed{(-3, 3)}$. Neither endpoint is included.