

Question 9

Let $S = \sum_{n=1}^{\infty} (-1)^n/n^2$ and $S_N = \sum_{n=1}^N (-1)^n/n^2$.

- (a) Give the alternating-series error bound.
- (b) Compare it with an Integral Test bound for the tail of $\sum 1/n^2$.
- (c) Evaluate both bounds at $N = 100$.

Question 9 – Solution**Step 1: Use cancellation.**

Since $1/n^2$ decreases to 0, the alternating estimate gives

$$|S - S_N| \leq \frac{1}{(N+1)^2}.$$

Step 2: Ignore signs and bound the absolute tail.

$$\sum_{n=N+1}^{\infty} \frac{1}{n^2} \leq \int_N^{\infty} \frac{dx}{x^2} = \frac{1}{N}.$$

This also bounds the signed error, but it discards cancellation and is therefore much weaker.

Step 3: Compare numerically.

At $N = 100$,

$$\frac{1}{101^2} = \frac{1}{10201} \approx 9.80 \times 10^{-5}, \quad \frac{1}{100} = 10^{-2}.$$

The alternating bound is over 100 times smaller. In general, it is order N^{-2} rather than N^{-1} .