

Question 8

Let $S = \sum_{n=1}^{\infty} ne^{-n}$ and $R_N = \sum_{n=N+1}^{\infty} ne^{-n}$. Use the Integral Test remainder theorem to give a two-sided bound for R_N .

Question 8 – Solution**Step 1: Check monotonicity.**For $f(x) = xe^{-x}$,

$$f'(x) = e^{-x}(1 - x) \leq 0 \quad (x \geq 1).$$

Thus f is positive, continuous, and decreasing on the needed interval.**Step 2: Find the antiderivative.**

Integration by parts gives

$$\int_A^\infty xe^{-x} dx = (A + 1)e^{-A}.$$

Step 3: Apply both remainder bounds.

$$\int_{N+1}^\infty xe^{-x} dx \leq R_N \leq \int_N^\infty xe^{-x} dx,$$

so

$$\boxed{(N + 2)e^{-(N+1)} \leq R_N \leq (N + 1)e^{-N}}.$$

The upper bound is a decreasing integral majorant for the full omitted tail; no extra first-term adjustment is needed under this standard formulation.