

Question 6

The Leibniz series is $\frac{\pi}{4} = \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1}$. If exactly M terms are used (indices 0 through $M-1$), find the least M certified by a next-term bound strictly below 10^{-4} .

Question 6 – Solution

Step 1: Identify the first omitted term.

With M terms, the final included index is $M - 1$, so the first omitted term has index M and magnitude $1/(2M + 1)$.

Step 2: Apply the alternating estimate.

$$|R| \leq \frac{1}{2M + 1}.$$

We need

$$\frac{1}{2M + 1} < 10^{-4} \iff 2M + 1 > 10,000 \iff M > 4999.5.$$

Thus the least integer is $\boxed{M = 5000}$.

Step 3: Verify minimality.

With 4999 terms the bound is $1/9999 > 10^{-4}$; with 5000 terms it is $1/10001 < 10^{-4}$. This large count illustrates the slow convergence of the Leibniz series.