

Question 4

Approximate $e^{-1} = \sum_{n=0}^{\infty} (-1)^n/n!$ using $S_N = \sum_{n=0}^N (-1)^n/n!$. Find the least final index N that guarantees correct rounding through six decimal places.

Question 4 – Solution**Step 1: Use the alternating remainder bound.**

The magnitudes $1/n!$ decrease to 0, so

$$|R_N| \leq \frac{1}{(N+1)!}.$$

The tolerance $|R_N| < 5 \times 10^{-7}$ suggests a candidate; correct rounding also requires a bracket within a single rounding interval.

Step 2: Find the first qualifying factorial.

$$9! = 362,880, \quad 10! = 3,628,800.$$

Hence

$$\frac{1}{9!} \approx 2.76 \times 10^{-6} > 5 \times 10^{-7}, \quad \frac{1}{10!} \approx 2.76 \times 10^{-7} < 5 \times 10^{-7}.$$

Therefore $N+1=10$, so $\boxed{N=9}$.

Step 3: Certify the rounding.

The alternating sign gives

$$0.3678791887 < S < S_9 + 1/10! < 0.3678794643.$$

Both endpoints lie in $[0.3678785, 0.3678795)$ and round to 0.367879. Also $S_8 \approx 0.3678819444$ and $S_7 \approx 0.3678571429$; the earlier even and odd partial sums lie still farther away. Hence no smaller final index gives this rounded value.

Conclusion.

Ten terms (indices 0 through 9) suffice. They give $S_9 \approx 0.3678791887$, which rounds to 0.367879, the same six-decimal value as e^{-1} .