

Question 3

Let $S = \sum_{n=1}^{\infty} 1/n^3$. Using the Integral Test upper remainder bound, find the least final index N certified to give absolute error $|R_N| < \frac{1}{2}10^{-4}$. Explain whether this tolerance alone guarantees identical rounding to four decimal places.

Question 3 – Solution

Step 1: Obtain an upper tail bound.

With $f(x) = x^{-3}$,

$$0 < R_N \leq \int_N^{\infty} x^{-3} dx = \left[-\frac{1}{2x^2} \right]_N^{\infty} = \frac{1}{2N^2}.$$

Step 2: Impose the absolute-error tolerance.

The requested error is below $0.00005 = 5 \times 10^{-5}$:

$$\frac{1}{2N^2} < 5 \times 10^{-5} \iff N^2 > 10,000 \iff N > 100.$$

Thus the least integer certified by this bound is $\boxed{N = 101}$.

Step 3: Check the strict endpoint.

At $N = 100$, the bound equals 5×10^{-5} , so it does not guarantee a strict error smaller than half a unit in the fourth decimal place. At $N = 101$, it does.

This absolute-error tolerance alone does not guarantee identical rounding: two nearby numbers can lie on opposite sides of a rounding threshold.