

Question 1

Let $S = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}$ and $S_N = \sum_{n=1}^N (-1)^{n-1}/n$.

- (a) State the Alternating Series Estimation Theorem for $R_N = S - S_N$.
- (b) Find the least final index N for which the next-term bound $1/(N+1)$ is strictly below 10^{-3} .

Question 1 – Solution**Step 1: Verify the theorem applies.**

The magnitudes $b_n = 1/n$ decrease to 0, so the alternating series converges and

$$|R_N| \leq b_{N+1} = \frac{1}{N+1}.$$

Step 2: Solve the strict inequality.

$$\frac{1}{N+1} < 10^{-3} \iff N+1 > 1000 \iff N > 999.$$

The least integer is $\boxed{N = 1000}$.

Step 3: Check minimality.

For $N = 999$, the displayed non-strict bound gives $|R_{999}| \leq 1/1000 = 10^{-3}$, so this bound is not strictly below the tolerance. (The actual remainder is strictly smaller than the next term.) For $N = 1000$, $|R_{1000}| \leq 1/1001 < 10^{-3}$.

Conclusion.

Use the first 1000 terms, ending at index 1000.