

Question 10

Determine whether $\sum_{n=1}^{\infty} \frac{3n^2 + 1}{n^3 - 2}$ converges or diverges. Identify the dominant powers and use Limit Comparison with an appropriate benchmark series.

Question 10 – Solution**Step 1: Identify the benchmark.**

The dominant-power quotient is

$$\frac{3n^2}{n^3} = \frac{3}{n},$$

so compare with the harmonic term $b_n = 1/n$. A finite number of initial terms, including the negative $n = 1$ term, does not affect convergence.

Step 2: Compute the limit.

For $n \geq 2$ both sequences are positive, and

$$\begin{aligned}\lim_{n \rightarrow \infty} \frac{a_n}{b_n} &= \lim_{n \rightarrow \infty} \frac{3n^2 + 1}{n^3 - 2} \cdot n \\ &= \lim_{n \rightarrow \infty} \frac{3 + 1/n^2}{1 - 2/n^3} = 3.\end{aligned}$$

The limit is finite and strictly positive.

Conclusion.

Since $\sum 1/n$ diverges, the given series **diverges** by the Limit Comparison Test. Dominant powers correctly predicted harmonic-size terms.