

Question 9

Determine whether $\sum_{n=2}^{\infty} \frac{\log n}{n^2}$ converges. Compare the Integral Test with Cauchy Condensation and identify the shorter calculation.

Question 9 – Solution**Step 1: Use the Integral Test.**

The function $f(x) = \log x/x^2$ is positive and continuous for $x \geq 2$. Also,

$$f'(x) = \frac{1 - 2 \log x}{x^3} < 0 \quad (x \geq 2).$$

Integration by parts gives

$$\int_2^\infty \frac{\log x}{x^2} dx = \left[-\frac{\log x + 1}{x} \right]_2^\infty = \frac{\log 2 + 1}{2} < \infty.$$

Thus the series converges.

Step 2: Compare with condensation.

Because the terms decrease,

$$2^k a_{2^k} = 2^k \frac{\log(2^k)}{(2^k)^2} = \frac{k \log 2}{2^k},$$

whose series converges by the Ratio Test. Condensation also works, but the integral is slightly shorter and matches the logarithm-over-power form directly.