

Question 8

Determine whether $\sum_{n=1}^{\infty} \left(1 + \frac{1}{n}\right)^{-n^2}$ converges. Explain why the Root Test is the natural first choice.

Question 8 – Solution

Step 1: Recognize the growing exponent.

Let $a_n = (1 + 1/n)^{-n^2}$. Taking an n th root cancels one power of n :

$$\sqrt[n]{a_n} = \left(1 + \frac{1}{n}\right)^{-n}.$$

Step 2: Use the exponential limit.

Since $(1 + 1/n)^n \rightarrow e$,

$$L = \lim_{n \rightarrow \infty} \sqrt[n]{a_n} = e^{-1} < 1.$$

Conclusion.

The series **converges** by the Root Test. This method is especially efficient because the summand is already a varying base raised to a multiple of n ; the root exposes its limiting geometric rate directly.