

Question 7

Classify $\sum_{n=1}^{\infty} \frac{\sin n}{n}$ as absolutely convergent, conditionally convergent, or divergent. Use bounded partial sums for the sine factor.

Question 7 – Solution**Step 1: Verify bounded partial sums.**

The finite identity

$$\sum_{n=1}^N \sin n = \frac{\sin(N/2) \sin((N+1)/2)}{\sin(1/2)}$$

shows that $\left| \sum_{n=1}^N \sin n \right| \leq 1/|\sin(1/2)|$.**Step 2: Apply Dirichlet's Test.**The sequence $1/n$ decreases to 0, and the partial sums of $\sin n$ are bounded. Hence $\sum \sin n/n$ converges.**Step 3: Check absolute convergence.**Since $|\sin n| \geq \sin^2 n$,

$$\sum \frac{|\sin n|}{n} \geq \sum \frac{\sin^2 n}{n} = \frac{1}{2} \sum \frac{1}{n} - \frac{1}{2} \sum \frac{\cos(2n)}{n}.$$

The cosine series converges by Dirichlet's Test while the harmonic series diverges, so the absolute-value series diverges.

Conclusion.The series is **conditionally convergent**.