

Question 6

Analyze $\sum_{n=1}^{\infty} \frac{n!}{(n+2)!}$.

- (a) Simplify the factorial quotient.
- (b) Decide whether telescoping or comparison is more informative.
- (c) Determine convergence and, if possible, the exact sum.

Question 6 – Solution

Step 1: Cancel factorials.

$$\frac{n!}{(n+2)!} = \frac{1}{(n+1)(n+2)}.$$

This already resembles a telescoping difference.

Step 2: Decompose and form partial sums.

$$\frac{1}{(n+1)(n+2)} = \frac{1}{n+1} - \frac{1}{n+2}.$$

Thus

$$\begin{aligned} S_N &= \sum_{n=1}^N \left(\frac{1}{n+1} - \frac{1}{n+2} \right) \\ &= \frac{1}{2} - \frac{1}{N+2}. \end{aligned}$$

Step 3: Pass to the limit.

$$\sum_{n=1}^{\infty} \frac{n!}{(n+2)!} = \lim_{N \rightarrow \infty} S_N = \boxed{\frac{1}{2}}.$$

Strategy note.

Comparison with $1/n^2$ proves convergence, but telescoping is better because it also gives the exact sum.