

Question 5

Determine whether $\sum_{n=1}^{\infty} \frac{1}{n^2 + \cos n}$ converges. For the tail $n \geq 2$, use only the bound $-1 \leq \cos n \leq 1$; no exact trigonometric values are needed.

Question 5 – Solution

Step 1: Ensure the terms are defined and positive.

For $n = 1$, $\cos 1 > 0$ because $0 < 1 < \pi/2$. For $n \geq 2$, $n^2 + \cos n \geq n^2 - 1 > 0$.

Step 2: Build a simple upper comparison.

For $n \geq 2$,

$$n^2 + \cos n \geq n^2 - 1 \geq \frac{1}{2}n^2.$$

Taking reciprocals reverses the denominator comparison:

$$0 < \frac{1}{n^2 + \cos n} \leq \frac{2}{n^2}.$$

Step 3: Apply the Comparison Test.

The series $\sum 2/n^2$ converges because it is a constant multiple of a p -series with $p = 2 > 1$.

Conclusion.

The given series **converges**. A direct comparison is more efficient than ratio or root tests because the bounded trigonometric perturbation does not change the dominant n^2 denominator.