

Question 3

Classify $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$ as absolutely convergent, conditionally convergent, or divergent. Use separate tests for ordinary and absolute convergence.

Question 3 – Solution

Step 1: Test the alternating series.

Put $b_n = 1/\sqrt{n}$. Then $b_n > 0$, $b_{n+1} < b_n$, and

$$\lim_{n \rightarrow \infty} b_n = 0.$$

Therefore $\sum (-1)^n b_n$ converges by the Alternating Series Test.

Step 2: Test absolute convergence.

$$\sum_{n=1}^{\infty} \left| \frac{(-1)^n}{\sqrt{n}} \right| = \sum_{n=1}^{\infty} \frac{1}{n^{1/2}}.$$

This is a divergent p -series because $p = 1/2 \leq 1$.

Conclusion.

The original series is **conditionally convergent**. One test establishes convergence; the second shows that convergence depends essentially on cancellation between alternating signs.