

Question 2

Determine whether $\sum_{n=2}^{\infty} \frac{1}{n \log n}$ converges or diverges. Explain why the Integral Test is especially well suited to this expression.

Question 2 – Solution**Step 1: Verify the hypotheses.**

Let $f(x) = 1/(x \log x)$ for $x \geq 2$. It is positive and continuous. Also,

$$f'(x) = -\frac{\log x + 1}{x^2(\log x)^2} < 0,$$

so f is decreasing.

Step 2: Evaluate the improper integral.

The integrand contains the differential $dx/x = d(\log x)$, suggesting $u = \log x$:

$$\int_2^b \frac{dx}{x \log x} = \int_{\log 2}^{\log b} \frac{du}{u} = \log(\log b) - \log(\log 2).$$

As $b \rightarrow \infty$, this tends to ∞ .

Conclusion.

The series **diverges** by the Integral Test. The substitution exposes the antiderivative $\log(\log x)$ immediately, which is why this test is more natural than a ratio or root test here.