

Question 1

Consider $\sum_{n=1}^{\infty} \frac{n}{3^n}$.

- (a) Choose an efficient convergence test and justify the choice.
- (b) Determine whether the series converges.
- (c) If possible, use a known power-series identity to find its exact sum.

Question 1 – Solution**Step 1: Prove convergence efficiently.**For $a_n = n/3^n$,

$$\frac{a_{n+1}}{a_n} = \frac{n+1}{3n} \longrightarrow \frac{1}{3} < 1.$$

The Ratio Test is quick and proves convergence.

Step 2: Choose a more informative method for the sum.

Begin with

$$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}, \quad |x| < 1.$$

Differentiate and multiply by x :

$$\sum_{n=1}^{\infty} nx^n = \frac{x}{(1-x)^2}.$$

At $x = 1/3$,

$$\sum_{n=1}^{\infty} \frac{n}{3^n} = \frac{1/3}{(1-1/3)^2} = \boxed{\frac{3}{4}}.$$

Strategy note.

The Ratio Test classifies the series; generating-function recognition gives both convergence and the exact value.