

Question 10

Let $c \in \mathbb{R}$. Determine exactly which values of c make

$$\sum_{n=1}^{\infty} c^{n^2}$$

converge. Apply the Root Test where decisive and inspect the boundary cases separately.

Question 10 – Solution

Step 1: Compute the root expression.

$$\sqrt[n]{|c^{n^2}|} = |c|^n.$$

Step 2: Separate the parameter cases.

- If $|c| < 1$, then $|c|^n \rightarrow 0 < 1$, so the series converges absolutely.
- If $|c| > 1$, then $|c|^n \rightarrow \infty$, and the terms c^{n^2} do not approach zero.
- If $|c| = 1$, the Root Test gives the boundary value 1. Directly, $|c^{n^2}| = 1$, so the terms again fail to approach zero.

Conclusion.

The series converges exactly when $|c| < 1$, equivalently $-1 < c < 1$. Both endpoints diverge.