

Question 9

Determine whether $\sum_{n=1}^{\infty} n^{-n}$ converges or diverges. Use the Root Test and explain why the exponent n makes this test especially efficient.

Question 9 – Solution**Step 1: Rewrite the term.**

We have $a_n = n^{-n} = (1/n)^n$.

Step 2: Take the n th root.

$$\sqrt[n]{|a_n|} = \sqrt[n]{\left(\frac{1}{n}\right)^n} = \frac{1}{n}.$$

Therefore

$$L = \lim_{n \rightarrow \infty} \frac{1}{n} = 0 < 1.$$

Conclusion.

The series **converges absolutely**. Rather than complicating the calculation, the exponent n is exactly canceled by the n th root. Moreover, for any $0 < r < 1$, eventually $1/n < r$, so $n^{-n} < r^n$ and the tail is dominated by a geometric series.