

Question 6

Analyze $\sum_{n=1}^{\infty} \left(\frac{n+1}{2n} \right)^n$ using the Root Test. State the root limit and the convergence classification.

Question 6 – Solution

Step 1: Take the n th root.

$$\sqrt[n]{\left|\left(\frac{n+1}{2n}\right)^n\right|} = \frac{n+1}{2n} = \frac{1}{2} + \frac{1}{2n}.$$

Step 2: Evaluate the limit.

$$L = \lim_{n \rightarrow \infty} \left(\frac{1}{2} + \frac{1}{2n}\right) = \frac{1}{2} < 1.$$

Conclusion.

The series **converges absolutely** by the Root Test. Although the base varies with n , it approaches $1/2$, so the terms eventually have geometric-scale decay. For example, once $n \geq 5$, the root is at most $3/5$, so $a_n \leq (3/5)^n$.