

Question 5

Determine whether

$$\sum_{n=2}^{\infty} \left(\frac{\log n}{n} \right)^n$$

converges or diverges. Compute the Root Test limit and interpret what a limit of zero implies.

Question 5 – Solution

Step 1: Take the n th root.

Since $n \geq 2$,

$$\sqrt[n]{\left| \left(\frac{\log n}{n} \right)^n \right|} = \frac{\log n}{n}.$$

Step 2: Evaluate the limit.

By l'Hopital's Rule for the corresponding continuous quotient,

$$\lim_{x \rightarrow \infty} \frac{\log x}{x} = \lim_{x \rightarrow \infty} \frac{1/x}{1} = 0.$$

Hence $L = 0 < 1$.

Conclusion.

The series **converges absolutely**. A root limit of zero means that for every fixed $0 < r < 1$, the terms eventually satisfy $a_n \leq r^n$; thus the tail is eventually dominated by a geometric series with any chosen ratio r .