

Question 4

Use the Root Test to determine whether $\sum_{n=1}^{\infty} \frac{n^5}{4^n}$ converges. Explain explicitly why the polynomial factor does not affect the final root limit.

Question 4 – Solution

Step 1: Take the n th root.

$$\sqrt[n]{\frac{n^5}{4^n}} = \frac{n^{5/n}}{4}.$$

Step 2: Evaluate the polynomial-root factor.

Take logarithms:

$$\log(n^{5/n}) = \frac{5 \log n}{n} \rightarrow 0.$$

Exponentiating gives $n^{5/n} \rightarrow e^0 = 1$.

Step 3: Apply the test.

$$L = \lim_{n \rightarrow \infty} \frac{n^{5/n}}{4} = \frac{1}{4} < 1.$$

Therefore the series **converges absolutely**. Under an n th root, any fixed power n^p becomes $n^{p/n} \rightarrow 1$, so the exponential denominator determines the result.