

Question 3

Find the radius and interval of convergence of

$$\sum_{n=1}^{\infty} (3^n + 2^n)^{1/n} x^n.$$

Use the Root Test for the interior and test both endpoints separately.

Question 3 – Solution

Step 1: Apply the root to the entire term.

For $a_n = (3^n + 2^n)^{1/n} x^n$,

$$\sqrt[n]{|a_n|} = (3^n + 2^n)^{1/n^2} |x|.$$

Write $3^n + 2^n = 3^n[1 + (2/3)^n]$. Then

$$(3^n + 2^n)^{1/n^2} = 3^{1/n} \left[1 + \left(\frac{2}{3} \right)^n \right]^{1/n^2} \longrightarrow 1.$$

Thus the Root Test limit is $L = |x|$: convergence holds for $|x| < 1$ and divergence for $|x| > 1$.

Step 2: Test the endpoints.

Since $(3^n + 2^n)^{1/n} \rightarrow 3$, the terms at $x = 1$ approach 3, and at $x = -1$ their magnitudes approach 3. Neither endpoint passes the nth-term test.

Conclusion.

The radius is $\boxed{R = 1}$ and the interval is $\boxed{(-1, 1)}$.