

Question 2

Analyze

$$\sum_{n=1}^{\infty} \left(1 + \frac{(-1)^n}{2} \right)^n.$$

- (a) Examine the even and odd terms separately.
- (b) Apply the nth-term divergence test.
- (c) Confirm the conclusion using the limsup form of the Root Test.

Question 2 – Solution**Step 1: Split by parity.**If $n = 2k$, then

$$a_{2k} = \left(1 + \frac{1}{2}\right)^{2k} = \left(\frac{3}{2}\right)^{2k} \rightarrow \infty.$$

If $n = 2k + 1$, then $a_{2k+1} = (1/2)^{2k+1} \rightarrow 0$.**Step 2: Apply the necessary condition.**

A convergent series must have $a_n \rightarrow 0$. The even subsequence is unbounded, so the full term sequence does not approach zero. Hence the series **diverges**.

Step 3: Check with roots.

$$\sqrt[n]{|a_n|} = 1 + \frac{(-1)^n}{2}, \quad \limsup_{n \rightarrow \infty} \sqrt[n]{|a_n|} = \frac{3}{2} > 1.$$

The limsup Root Test gives the same result; an ordinary root limit does not exist.