

Question 1

Consider

$$\sum_{n=1}^{\infty} \frac{(2n+1)^n}{5^n n^n}.$$

- (a) Rewrite the summand as a single quantity raised to the n th power.
- (b) Apply the Root Test and classify the series.

Question 1 – Solution**Step 1: Rewrite the term.**

Let

$$a_n = \frac{(2n+1)^n}{5^n n^n} = \left(\frac{2n+1}{5n} \right)^n = \left(\frac{2}{5} + \frac{1}{5n} \right)^n.$$

Step 2: Take the n th root.Since $a_n > 0$,

$$\sqrt[n]{|a_n|} = \frac{2n+1}{5n} = \frac{2}{5} + \frac{1}{5n}.$$

Therefore $L = \lim_{n \rightarrow \infty} \sqrt[n]{|a_n|} = 2/5 < 1$.**Conclusion.**

The series **converges absolutely** by the Root Test. The slowly varying $1/(5n)$ correction disappears in the limit, leaving geometric-scale decay with factor $2/5$.