

Question 9

Determine whether $\sum_{n=1}^{\infty} \frac{n^n}{(n!)^2}$ converges or diverges. Reduce the consecutive-term ratio completely before taking its limit.

Question 9 – Solution

Step 1: Form the ratio.

$$\begin{aligned}\frac{a_{n+1}}{a_n} &= \frac{(n+1)^{n+1}}{((n+1)!)^2} \frac{(n!)^2}{n^n} = \frac{(n+1)^{n-1}}{n^n} \\ &= \frac{1}{n} \left(1 + \frac{1}{n}\right)^{n-1}.\end{aligned}$$

Step 2: Separate the limiting factors.

We have $(1 + 1/n)^{n-1} \rightarrow e$ and $1/n \rightarrow 0$. Therefore $L = 0$.

Conclusion.

Since $L = 0 < 1$, the positive-term series **converges**. The simplified ratio resolves the competition between n^n and $(n!)^2$ without guesswork.