

Question 8

Analyze $\sum_{n=1}^{\infty} \frac{2^{n^2}}{3^{n^2}}$. Rewrite its term using a single base, compute the consecutive-term ratio, and classify the series.

Question 8 – Solution

Step 1: Combine the powers.

$$a_n = (2/3)^{n^2}.$$

Step 2: Compute the ratio.

$$\frac{a_{n+1}}{a_n} = \left(\frac{2}{3}\right)^{(n+1)^2 - n^2} = \left(\frac{2}{3}\right)^{2n+1}.$$

Because $0 < 2/3 < 1$, this expression tends to 0.

Conclusion.

The ratio limit is $L = 0 < 1$, so the series **converges**. Its quadratic exponent makes the terms decay faster than those of $\sum (2/3)^n$.