

Question 7

Determine whether $\sum_{n=1}^{\infty} \frac{n!}{n^{10}}$ converges or diverges.

- (a) Simplify a_{n+1}/a_n .
- (b) Use its limiting behavior to justify the conclusion.
- (c) Explain why the nth-term test also applies.

Question 7 – Solution

Step 1: Compute the ratio.

$$\begin{aligned}\frac{a_{n+1}}{a_n} &= \frac{(n+1)!}{(n+1)^{10}} \frac{n^{10}}{n!} = (n+1) \left(\frac{n}{n+1} \right)^{10} \\ &= n \left(1 + \frac{1}{n} \right)^{-9}.\end{aligned}$$

Step 2: Evaluate its behavior.

The factor $(1 + 1/n)^{-9} \rightarrow 1$, while $n \rightarrow \infty$. Hence $L = \infty > 1$.

Conclusion.

The series **diverges**. Its terms eventually grow rather than approach zero, so the nth-term divergence test confirms the result. A factorial eventually dominates every fixed power n^{10} .