

**Question 6**

For  $\sum_{n=0}^{\infty} \frac{x^n}{n!}$ , where  $x \in \mathbb{R}$ :

- (a) Apply the Ratio Test to the absolute values.
- (b) Find every real  $x$  for which the series converges.
- (c) State the exact sum.

**Question 6 – Solution****Step 1: Treat  $x$  as fixed.**For  $a_n = x^n/n!$ ,

$$\left| \frac{a_{n+1}}{a_n} \right| = \frac{|x|}{n+1}.$$

For  $x = 0$  the series plainly converges; the same final limit covers every fixed nonzero  $x$ .**Step 2: Take the limit.**

$$L = \lim_{n \rightarrow \infty} \frac{|x|}{n+1} = 0 < 1 \quad \text{for every fixed } x \in \mathbb{R}.$$

**Conclusion.**

The series converges absolutely for all real  $x$ , so its interval of convergence is  $(-\infty, \infty)$ . By the exponential-series identity,

$$\boxed{\sum_{n=0}^{\infty} \frac{x^n}{n!} = e^x}.$$