

Question 4

Analyze $\sum_{n=0}^{\infty} \frac{(2n)!}{(n!)^2 5^n}$. Compute the consecutive-term ratio carefully, apply the Ratio Test, and state the convergence classification.

Question 4 – Solution

Step 1: Name the term.

Let $a_n = (2n)! / [(n!)^2 5^n]$.

Step 2: Expand only the new factorial factors.

$$\begin{aligned}\frac{a_{n+1}}{a_n} &= \frac{(2n+2)!}{((n+1)!)^2 5^{n+1}} \frac{(n!)^2 5^n}{(2n)!} \\ &= \frac{(2n+2)(2n+1)}{5(n+1)^2} = \frac{2(2n+1)}{5(n+1)}.\end{aligned}$$

Step 3: Take the limit.

$$L = \lim_{n \rightarrow \infty} \frac{2(2n+1)}{5(n+1)} = \frac{4}{5} < 1.$$

Conclusion.

The positive-term series **converges**. The factorial expression is $\binom{2n}{n}$, but no approximation is needed.