

Question 3

Determine whether $\sum_{n=1}^{\infty} \frac{n^2}{2^n}$ converges or diverges.

- (a) Compute and simplify a_{n+1}/a_n .
- (b) Apply the Ratio Test.
- (c) Explain how the limit compares polynomial and exponential growth.

Question 3 – Solution

Step 1: Compute the exact ratio.

$$\frac{a_{n+1}}{a_n} = \frac{(n+1)^2}{2^{n+1}} \frac{2^n}{n^2} = \frac{1}{2} \left(1 + \frac{1}{n}\right)^2.$$

Step 2: Evaluate the limit.

$$L = \frac{1}{2}(1)^2 = \frac{1}{2} < 1.$$

Conclusion.

The series **converges absolutely**. Its consecutive terms eventually behave as though multiplied by $1/2$. The polynomial factor changes the ratio by a factor tending to 1, while 2^n supplies the decisive factor $1/2$.