

Question 2

Consider $\sum_{n=0}^{\infty} \frac{3^n}{n!}$.

- (a) Apply the Ratio Test and classify the series.
- (b) Recognize the resulting series and find its exact sum.

Question 2 – Solution

Step 1: Form the ratio.

With $a_n = 3^n/n!$,

$$\frac{a_{n+1}}{a_n} = \frac{3^{n+1}}{(n+1)!} \frac{n!}{3^n} = \frac{3}{n+1}.$$

Step 2: Take the limit.

$L = \lim_{n \rightarrow \infty} 3/(n+1) = 0 < 1$. The Ratio Test proves absolute convergence.

Step 3: Identify the sum.

Since $e^x = \sum_{n=0}^{\infty} x^n/n!$, substituting $x = 3$ gives

$$\boxed{\sum_{n=0}^{\infty} \frac{3^n}{n!} = e^3}.$$