

Question 1

Consider $\sum_{n=1}^{\infty} \frac{n!}{3^n}$.

- (a) Compute a_{n+1}/a_n .
- (b) Decide whether the series converges.
- (c) Explain what the calculation says about factorial growth compared with 3^n .

Question 1 – Solution**Step 1: Identify the term.**

Let $a_n = n!/3^n$. Its terms are positive, so absolute values do not change the ratio.

Step 2: Simplify before taking the limit.

$$\frac{a_{n+1}}{a_n} = \frac{(n+1)!}{3^{n+1}} \frac{3^n}{n!} = \frac{n+1}{3}.$$

Step 3: Interpret the ratio.

Thus $L = \lim_{n \rightarrow \infty} (n+1)/3 = \infty > 1$. The terms eventually increase; indeed, $a_{n+1} > a_n$ for $n > 2$. Therefore a_n cannot approach 0.

Conclusion.

The series **diverges** by the Ratio Test (and by the nth-term test). The unbounded ratio shows that $n!$ eventually grows faster than 3^n .