

Question 2

Let $a_n = (-1)^n \left(1 + \frac{1}{n}\right)$ for $n \geq 1$.

- (a) Show that (a_n) is bounded.
- (b) Find the limits of the even and odd subsequences (a_{2k}) and (a_{2k-1}) .
- (c) Use those limits to decide whether (a_n) converges and to evaluate the claim, “Every bounded sequence converges.”

Question 2 – Solution**Step 1: Prove boundedness.**

For $n \geq 1$, $1 < 1 + 1/n \leq 2$. Since $|(-1)^n| = 1$,

$$|a_n| = 1 + \frac{1}{n} \leq 2.$$

Thus $-2 \leq a_n \leq 2$, and the sequence is bounded.

Step 2: Analyze the even subsequence.

For even indices,

$$a_{2k} = 1 + \frac{1}{2k} \rightarrow 1.$$

Step 3: Analyze the odd subsequence.

For odd indices,

$$a_{2k-1} = -1 - \frac{1}{2k-1} \rightarrow -1.$$

Step 4: Apply the subsequence criterion.

If (a_n) converged to L , every subsequence would also converge to L . The even and odd subsequences instead approach 1 and -1 , respectively. Since these limits differ, (a_n) diverges.

Step 5: Evaluate the student's claim.

This bounded divergent sequence is a counterexample to the claim that every bounded sequence converges. Boundedness together with monotonicity, however, would guarantee convergence by the Monotone Convergence Theorem.